



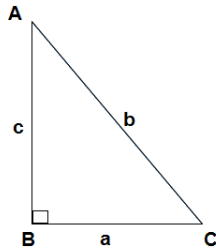
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GRADE 10 CAPS Curriculum

10.9 Trigonometry

1.1 Define the trigonometric ratios $\sin \theta$, $\cos \theta$ and $\tan \theta$ using right-angled triangle.

(a)

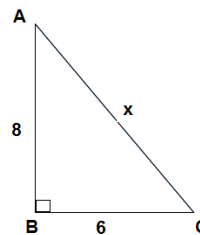


$$\cos A = \dots$$

$$\sin C = \dots$$

$$\tan A = \dots$$

(b)

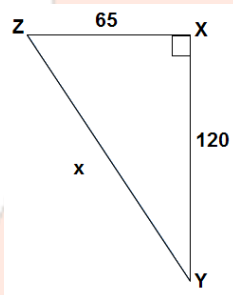


$$\sin A = \dots$$

$$\tan C = \dots$$

$$\cos C = \dots$$

(c)



$$\sin Z = \dots$$

$$\sin Y = \dots$$

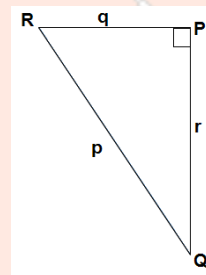
$$\cos Z = \dots$$

$$\cos Y = \dots$$

$$\tan Z = \dots$$

$$\tan Y = \dots$$

(d)



$$\sin Q = \dots$$

$$\tan R = \dots$$

$$\cos Q = \dots$$

1.2 Extend the definitions of $\sin \theta$, $\cos \theta$ and $\tan \theta$ for $0^\circ \leq \theta \leq 360^\circ$.

(a) $\cos 100^\circ$

(b) $\tan 210^\circ$

(c) $\sin 300^\circ$

(d) $\tan 135^\circ$

(e) $\sin 315^\circ$

(f) $\cos 120^\circ$

(g) $\sin 240^\circ$

(h) $\cos 225^\circ$

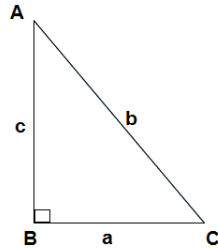
(i) $\tan 150^\circ$

(j) $\sin 135^\circ$

1.3 Define the reciprocals of the trigonometric ratios $\operatorname{cosec} \theta$, $\sec \theta$ and $\cot \theta$, using right-angled triangles.



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$\sin A = \dots$	$\operatorname{cosec} C = \dots$
$\cos A = \dots$	$\sec C = \dots$
$\tan A = \dots$	$\cot C = \dots$
$\operatorname{cosec} A = \dots$	$\sin C = \dots$
$\sec A = \dots$	$\cos C = \dots$
$\cot A = \dots$	$\tan C = \dots$

1.4 Derive values of the trigonometric ratios for the special cases (without using a calculator).

(a) $\frac{\tan 225^\circ \cdot \sin 135^\circ \cdot \tan 300^\circ}{\cos 315^\circ \cdot \cos 225^\circ \cdot \cos 150^\circ}$

(b) $\frac{\tan 120^\circ}{\tan 330^\circ}$

(c) $\sin 60^\circ \cdot \cos 30^\circ \cdot \tan 60^\circ$

(d) $\sin 30^\circ \cdot \tan 45^\circ \cdot \cos 45^\circ$

(e) $\frac{\tan 120^\circ \cdot \cos 210^\circ}{\sin 240^\circ \cdot \sin 240^\circ}$

(f) $\frac{\cos 330^\circ}{\cos 225^\circ \cdot \cos 315^\circ \cdot \tan 225^\circ}$

1.5 Solve two-dimensional problems involving right-angled triangles.

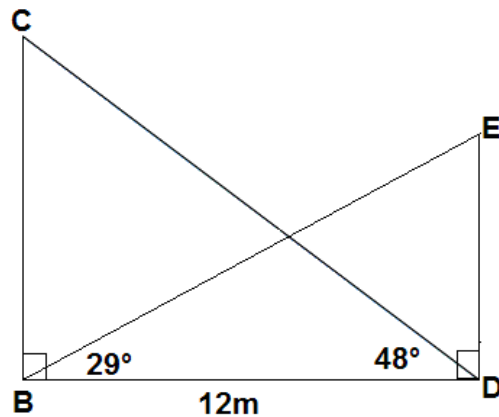
(a) The length of a mast is 8.5m, and the length of the shadow of the mast is 7.25m. Calculate the angle of elevation of the sun at the particular moment.



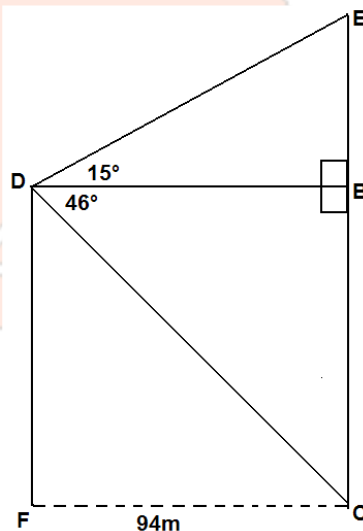
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(b) The angle of elevation of a glider according to a woman on the ground is 43° . If the glider is 2340m from the woman, calculate the altitude of the glider.

(c) Two towers are 12m apart. From B the angle of elevation to DE is 29° and from D the angle of elevation to BC is 48° . Calculate the difference in the heights of the towers.



(d) A building (DF) and a tower (CE) are 94m apart. From the roof of the building the angle of elevation to the top of the tower is 15° and the angle of depression to the bottom of the tower is 46° . Calculate the height of the tower.



1.6 Solve simple trigonometric equations for angles between 0° and 90° .

(a) $\sin 51^\circ = \cos \beta$, β an acute angle.

(b) $\cos 33^\circ = \sin \alpha$

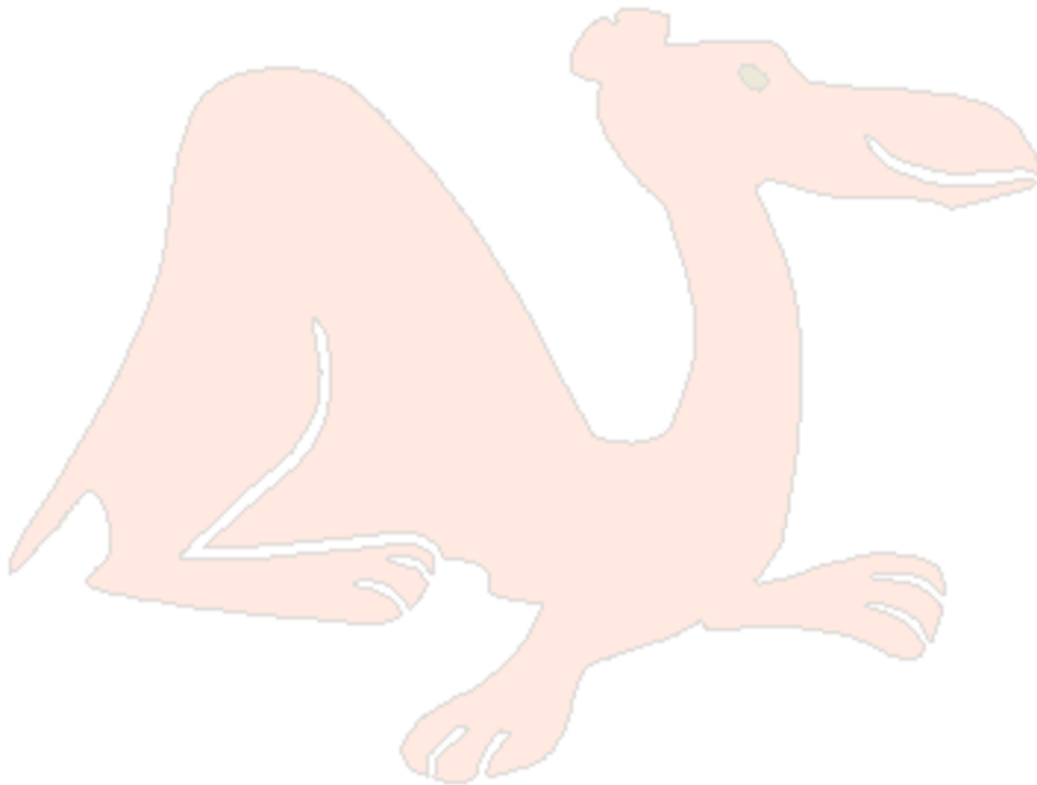


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- (c) $\sin 75^\circ = \cos 3\theta$
- (d) $\cos 4\alpha = \sin 5\alpha$
- (e) $\cos(\beta - 43^\circ) = \sin 65^\circ$
- (f) $\sin(\theta + 54^\circ) = \cos(\theta - 8^\circ)$

1.7 Use diagrams to determine the numerical values of ratios for angles from 0° and 360° .

- (a) If $17\sin A = 15$, $0^\circ \leq A \leq 90^\circ$, determine $\tan A$.
- (b) If $9\tan \beta = 40$ and β is an acute angle, determine $\sin \beta$.
- (c) If $6\sin \alpha - 5 = 0$ and $\alpha \in [90^\circ; 180^\circ]$, determine $\cos \alpha$.
- (d) If $-5\cos \beta - 4 = 0$ and $\beta \in [180^\circ; 270^\circ]$, determine $\sin \beta$.
- (e) If $5\sin \theta - 4 = 0$ and $90^\circ \leq \theta \leq 180^\circ$, determine $\cos \theta$.



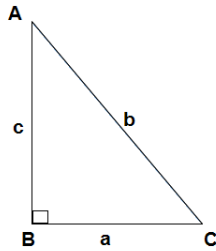


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MEMO

1.1 Define the trigonometric ratios $\sin \theta$, $\cos \theta$ and $\tan \theta$ using right-angled triangle. [7.2.1.1]

(a)

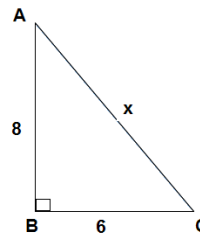


$$\cos A = \frac{c}{b}$$

$$\sin C = \frac{c}{b}$$

$$\tan A = \frac{a}{c}$$

(b)

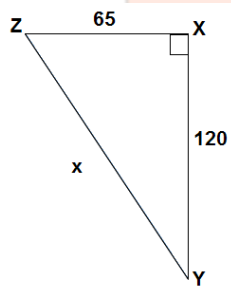


$$\sin A = \frac{6}{x}$$

$$\tan C = \frac{8}{6}$$

$$\cos C = \frac{6}{x}$$

(c)

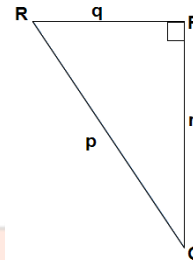


$$\sin Z = \frac{120}{x} ; \sin Y = \frac{65}{x}$$

$$\cos Z = \frac{65}{x} ; \cos Y = \frac{120}{x}$$

$$\tan Z = \frac{120}{65} ; \tan Y = \frac{65}{120}$$

(d)



$$\sin Q = \frac{q}{p}$$

$$\tan R = \frac{r}{q}$$

$$\cos Q = \frac{r}{p}$$

1.2 Extend the definitions of $\sin \theta$, $\cos \theta$ and $\tan \theta$ for $0^\circ \leq \theta \leq 360^\circ$. [7.4.2.2; 7.4.2.3]

(a) $\cos 100^\circ = -\cos 80^\circ$

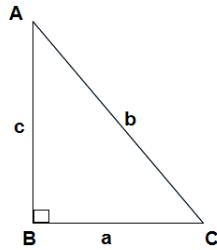
(b) $\tan 210^\circ = \tan 30^\circ$



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- (c) $\sin 300^\circ = -\sin 60^\circ$ (d) $\tan 135^\circ = -\tan 45^\circ$
(e) $\sin 315^\circ = -\sin 45^\circ$ (f) $\cos 120^\circ = -\cos 60^\circ$
(g) $\sin 240^\circ = -\sin 60^\circ$ (h) $\cos 225^\circ = -\cos 45^\circ$
(i) $\tan 150^\circ = -\tan 30^\circ$ (j) $\sin 135^\circ = \sin 45^\circ$

1.3 Define the reciprocals of the trigonometric ratios cosec θ , sec θ and cot θ , using right-angled triangles. [7.2.1.3; 7.2.1.4; 7.2.1.5; 7.2.1.2]



$\sin A = \frac{a}{b}$	$\operatorname{cosec} C = \frac{b}{c}$
$\cos A = \frac{c}{b}$	$\sec C = \frac{b}{a}$
$\tan A = \frac{a}{c}$	$\cot C = \frac{a}{c}$
$\operatorname{cosec} A = \frac{b}{a}$	$\sin C = \frac{c}{b}$
$\sec A = \frac{b}{c}$	$\cos C = \frac{a}{b}$
$\cot A = \frac{c}{a}$	$\tan C = \frac{c}{a}$

1.4 Derive values of the trigonometric ratios for the special cases (without using a calculator). [7.3.2.1; 7.3.2.3; 7.3.1.5; 7.3.1.1]



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(a)

$$\begin{aligned} & \frac{\tan 225^\circ \cdot \sin 135^\circ \cdot \tan 300^\circ}{\cos 315^\circ \cdot \cos 225^\circ \cdot \cos 150^\circ} \\ &= \frac{\tan 45^\circ \cdot \sin 45^\circ \cdot (-\tan 60^\circ)}{\cos 45^\circ \cdot (-\cos 45^\circ) \cdot (-\cos 30^\circ)} \\ &= \frac{1 \cdot \frac{1}{\sqrt{2}} \cdot (-\sqrt{3})}{\frac{1}{\sqrt{2}} \cdot (-\frac{1}{\sqrt{2}}) \cdot (-\frac{\sqrt{3}}{2})} \\ &= -2\sqrt{2} \end{aligned}$$

(b)

$$\begin{aligned} & \frac{\tan 120^\circ}{\tan 330^\circ} \\ &= \frac{-\tan 60^\circ}{-\tan 30^\circ} \\ &= \frac{\sqrt{3}}{\frac{1}{\sqrt{3}}} \\ &= 3 \end{aligned}$$

(c)

$$\begin{aligned} & \sin 60^\circ \cdot \cos 30^\circ \cdot \tan 60^\circ \\ &= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} \cdot \sqrt{3} \\ &= \frac{3\sqrt{3}}{4} \end{aligned}$$

(d)

$$\begin{aligned} & \sin 30^\circ \cdot \tan 45^\circ \cdot \cos 45^\circ \\ &= \frac{1}{2} \cdot 1 \cdot \frac{1}{\sqrt{2}} \\ &= \frac{1}{2\sqrt{2}} \end{aligned}$$



(e)

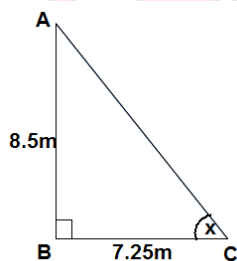
$$\begin{aligned} & \frac{\tan 120^\circ \cdot \cos 210^\circ}{\sin 240^\circ \cdot \sin 240^\circ} \\ &= \frac{(-\tan 60^\circ) \cdot (-\cos 30^\circ)}{(-\sin 60^\circ) \cdot (-\sin 60^\circ)} \\ &= \frac{\sqrt{3} \cdot \frac{\sqrt{3}}{2}}{\left(\frac{\sqrt{3}}{2}\right)^2} \\ &= 2 \end{aligned}$$

(f)

$$\begin{aligned} & \frac{\cos 330^\circ}{\cos 225^\circ \cdot \cos 315^\circ \cdot \tan 225^\circ} \\ &= \frac{\cos 30^\circ}{(-\cos 45^\circ) \cdot \cos 45^\circ \cdot \tan 45^\circ} \\ &= \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \cdot 1} \\ &= -\sqrt{3} \end{aligned}$$

1.5 Solve two-dimensional problems involving right-angled triangles. [7.7.1.1; 7.7.1.2; 7.7.1.3]

(a) The length of a mast is 8.5m, and the length of the shadow of the mast is 7.25m. Calculate the angle of elevation of the sun at the particular moment.



$$\frac{AB}{BC} = \tan x$$

$$\frac{8.5}{7.25} = \tan x$$

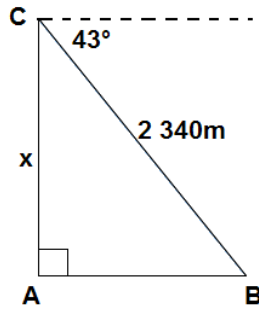
$$x = \tan^{-1}\left(\frac{8.5}{7.25}\right)$$

$$x = 49.5^\circ$$



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(b) The angle of elevation of a glider according to a woman on the ground is 43° . If the glider is 2340m from the woman, calculate the altitude of the glider.



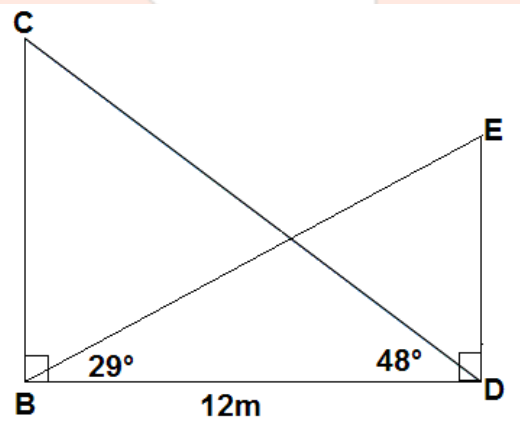
$$\frac{AC}{BC} \sin x$$

$$\frac{x}{2340} = \sin 43^\circ$$

$$x = 2340 \times \sin 43^\circ$$

$$x = 1595.9m$$

(c) Two towers are 12m apart. From B the angle of elevation to DE is 29° and from D the angle of elevation to BC is 48° . Calculate the difference in the heights of the towers.



$\triangle BCD$:

$$\frac{BC}{BD} = \tan 48^\circ$$

$$\frac{x}{12} = \tan 48^\circ$$

$$x = 12 \times \tan 48^\circ$$

$$x = 13.33m$$

$\triangle BDE$:

$$\frac{DE}{BD} = \tan 29^\circ$$

$$\frac{y}{12} = \tan 29^\circ$$

$$y = 12 \times \tan 29^\circ$$

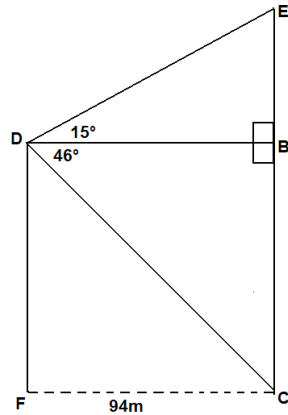
$$y = 6.65m$$

$$BC - DE = 13.33 - 6.65 = 6.68m$$



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(d) A building (DF) and a tower (CE) are 94m apart. From the roof of the building the angle of elevation to the top of the tower is 15° and the angle of depression to the bottom of the tower is 46° . Calculate the height of the tower.



$\triangle BCD$:

$$\frac{BC}{DB} = \tan 46^\circ$$

$$\frac{x}{94} = \tan 46^\circ$$

$$x = 94 \times \tan 46^\circ$$

$$x = 97.34m$$

$\triangle BDE$:

$$\frac{BE}{DB} = \tan 15^\circ$$

$$\frac{y}{94} = \tan 15^\circ$$

$$y = 94 \times \tan 15^\circ$$

$$y = 25.19m$$

$$EC = 97.34 + 25.19 = 122.53m$$

1.6 Solve simple trigonometric equations for angles between 0° and 90° . [7.6.2.1; 7.6.2.3; 7.6.2.5]

(a) $\sin 51^\circ = \cos \beta$, β an acute angle.

$$\sin 51^\circ = \cos \beta$$

$$\sin 51^\circ = \sin(90^\circ - \beta)$$

$$\therefore 51^\circ = 90^\circ - \beta$$

$$\therefore \beta = 90^\circ - 51^\circ$$

$$\therefore \beta = 39^\circ$$

(b) $\cos 33^\circ = \sin \alpha$



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$$\cos 33^\circ = \sin \alpha$$

$$\cos 33^\circ = \cos(90^\circ - \alpha)$$

$$\therefore 33^\circ = 90^\circ - \alpha$$

$$\therefore \alpha = 90^\circ - 33^\circ$$

$$\therefore \alpha = 57^\circ$$

(c) $\sin 75^\circ = \cos 3\theta$

$$\sin 75^\circ = \cos 3\theta$$

$$\sin 75^\circ = \sin(90^\circ - 3\theta)$$

$$\therefore 75^\circ = 90^\circ - 3\theta$$

$$\therefore 3\theta = 90^\circ - 75^\circ$$

$$\therefore 3\theta = 15^\circ$$

$$\therefore \theta = 5^\circ$$

(d) $\cos 4\alpha = \sin 5\alpha$

$$\cos 4\alpha = \sin 5\alpha$$

$$\cos 4\alpha = \cos(90^\circ - 5\alpha)$$

$$\therefore 4\alpha = 90^\circ - 5\alpha$$

$$\therefore 9\alpha = 90^\circ$$

$$\therefore \alpha = 10^\circ$$

(e) $\cos(\beta - 43^\circ) = \sin 65^\circ$

$$\cos(\beta - 43^\circ) = \sin 65^\circ$$

$$\cos(\beta - 43^\circ) = \cos(90^\circ - 65^\circ)$$

$$\therefore \beta - 43^\circ = 90^\circ - 65^\circ$$

$$\therefore \beta = 68^\circ$$

(f) $\sin(\theta + 54^\circ) = \cos(\theta - 8^\circ)$

$$\sin(\theta + 54^\circ) = \cos(\theta - 8^\circ)$$

$$\sin(\theta + 54^\circ) = \sin(90^\circ - (\theta - 8^\circ))$$

$$\sin(\theta + 54^\circ) = \sin(90^\circ - \theta + 8^\circ)$$

$$\therefore \theta + 54^\circ = 90^\circ - \theta + 8^\circ$$

$$\therefore 2\theta = 90^\circ - 54^\circ + 8^\circ$$

$$\therefore 2\theta = 44^\circ$$

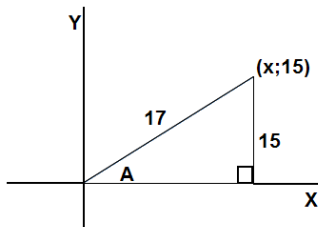
$$\therefore \theta = 22^\circ$$



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1.7 Use diagrams to determine the numerical values of ratios for angles from 0° and 360° . [7.6.3.1; 7.6.3.3; 7.6.3.5; 7.6.5.1]

(a) If $17\sin A = 15$, $0^\circ \leq A \leq 90^\circ$, determine $\tan A$.



$$x^2 + y^2 = r^2$$

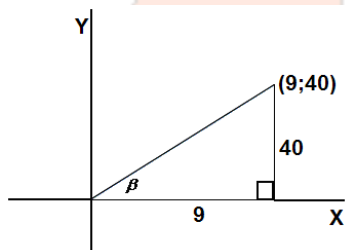
$$x^2 + (15)^2 = (17)^2$$

$$x^2 = 64$$

$$x = 8$$

$$\therefore \tan A = \frac{15}{8}$$

(b) If $9\tan \beta = 40$ and β is an acute angle, determine $\sin \beta$.



$$x^2 + y^2 = r^2$$

$$(9)^2 + (40)^2 = r^2$$

$$81 + 1600 = r^2$$

$$r^2 = 1681$$

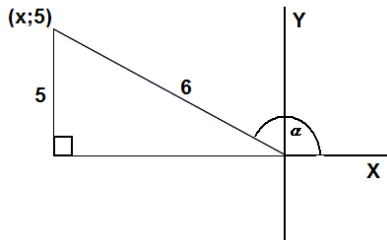
$$r = 41$$

$$\therefore \sin \beta = \frac{40}{41}$$

(c) If $6\sin \alpha - 5 = 0$ and $\alpha \in [90^\circ; 180^\circ]$, determine $\cos \alpha$.



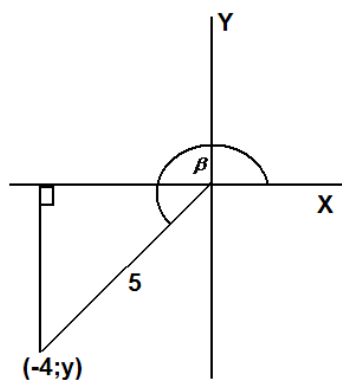
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$$\begin{aligned}x^2 + y^2 &= r^2 \\x^2 + (5)^2 &= (6)^2 \\x^2 &= 36 - 25 \\x &= -\sqrt{11}\end{aligned}$$

$$\therefore \cos \alpha = \frac{-\sqrt{11}}{6}$$

(d) If $-5\cos \beta - 4 = 0$ and $\beta \in [180^\circ; 270^\circ]$, determine $\sin \beta$.



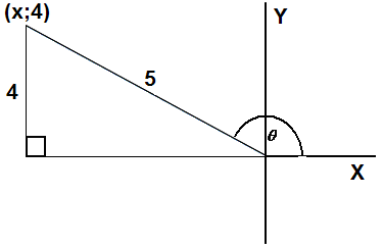
$$\begin{aligned}x^2 + y^2 &= r^2 \\(-4)^2 + y^2 &= (5)^2 \\16 + y^2 &= 25 \\y &= -3\end{aligned}$$

$$\therefore \sin \beta = \frac{-3}{5}$$

(e) If $5\sin \theta - 4 = 0$ and $90^\circ \leq \theta \leq 180^\circ$, determine $\cos \theta$.



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$x^2 + y^2 = r^2$
 $x^2 + (4)^2 = (5)^2$
 $x^2 + 16 = 25$
 $x = -3$

$\therefore \cos \theta = \frac{-3}{5}$

