GRADE 10_CAPS Curriculum

10.8 Analytical geometry

1.1 Distance formula

(a) Calculate the length of GH, where G is (-8;-1) and H is (-5;8).
(b) Calculate the length of AB, where A is (7;1) and B is (-6;7).
(c) Calculate the length of ST, where S is (4;6) and T is (-4;-6).
(d) Calculate the length of FG, where F is (-5;4) and g is (-1;7).
(e) Find the value(s) of q if the distance between L(q;3) and M(1;-1) is $\sqrt{17}$.
(f) Find the value(s) of z if the distance between D(-8;z) and E(-3;-3) is $\sqrt{26}$.
(g) Find the value(s) of r if the distance between C(-5;r) and D(-1;-2) is $\sqrt{20}$.

1.2 Gradient between two points (parallel and/or perpendicular lines)

(a) If A(0;-4) , B(-5;-5) , C(1;4) and D(2;9) are given, calculate the gradients of AB and CD and state whether they are parallel or perpendicular.

(b) If K(5;-4) , L(0;-8) , M(-1;-5) and N(3;-10) are given, calculate the gradients of KL and MN and state whether they are parallel or perpendicular.

(c) If I(1;-5), J(-3;0), K(g;-4) and L(-3;1) are given, calculate the value of g if IJ and KL are parallel.

(d) G(1;-4) , H(0;-2) , I(3 ;q) and J(1;-1) are given, calculate the value of q if GH and IJ are perpendicular.

1.3 Midpoint theorem

(a) Find the midpoint of the line joining the points D(-10;-10) and E(5;5).

(b) Find the midpoint of the line joining the points J(2;-6) and K(-3;-4).

(c) Find the midpoint of the line joining the points M(-1;-4) and N(-6;-2).

(d) Find the coordinates of the midpoint of the line joining (-p-9k;6z-7m) and (-7p+7k;2z-7m).

(e) The point (1;1) lies at the centre of a line joining (-4;7) and (q;-5). Find q.

(f) The point (5;-1) lies at the centre of a line joining (7;1) and (y;-3). Find y.

(g) The point (-7;0) lies at the centre of a line joining (-9;-6) and (-5;p). Find p.

MEMO

1.1 Distance formula [8.8.1.1; 8.8.1.2; 8.8.1.3]

(a) G (-8;-1) and H (-5;8)

$$GH = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$GH = \sqrt{(-5+8)^2 + (8+1)^2}$$
$$GH = \sqrt{3^2 + 9^2}$$
$$GH = \sqrt{9+81}$$
$$GH = \sqrt{90}$$
$$GH = 3\sqrt{10}$$

(b) A(7;1) and B(-6;7)

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$AB = \sqrt{(-6-7)^2 + (7-1)^2}$$
$$AB = \sqrt{(-13)^2 + 6^2}$$
$$AB = \sqrt{169+36}$$
$$AB = \sqrt{205}$$

(c) S (4;6) and T(-4;-6)

$$ST = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$ST = \sqrt{(-4-4)^2 + (-6-6)^2}$$
$$ST = \sqrt{(-8)^2 + (-12)^2}$$
$$ST = \sqrt{64+144}$$
$$ST = \sqrt{208}$$
$$ST = 4\sqrt{13}$$

(d) F(-5;4) and G(-1;7)

$$FG = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$FG = \sqrt{(-1+5)^2 + (7-4)^2}$$
$$FG = \sqrt{(4)^2 + (3)^2}$$
$$FG = \sqrt{16+9}$$
$$FG = \sqrt{25}$$
$$FG = 5$$

(e) L(q;3) and M(1;-1), LM = $\sqrt{17}$

$$LM = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$\sqrt{17} = \sqrt{(1-q)^2 + (-1-3)^2}$$
$$\sqrt{17} = \sqrt{1-2q+q^2 + (-4)^2}$$
$$17 = q^2 - 2q + 17$$
$$0 = q^2 - 2q$$
$$0 = q(q-2)$$
$$q = 0; q = 2$$

(f) D(-8;z) and E(-3;-3), DE = $\sqrt{26}$

$$DE = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$\sqrt{26} = \sqrt{(-3+8)^2 + (-3-z)^2}$$
$$\sqrt{26} = \sqrt{(5)^2 + 9 + 6z + z^2}$$
$$26 = z^2 + 6z + 34$$
$$0 = z^2 + 6z + 8$$
$$0 = (z+4)(z+2)$$
$$z = -4; z = -2$$

(g) C(-5;r) and D(-1;-2), CD = $\sqrt{20}$

$$CD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$\sqrt{20} = \sqrt{(-1+5)^2 + (-2-r)^2}$$
$$\sqrt{20} = \sqrt{(4)^2 + 4 + 4r + r^2}$$
$$20 = r^2 + 4r + 20$$
$$0 = r^2 + 4r$$
$$0 = r(r+4)$$
$$r = 0; r = -4$$

1.2 Gradient between two points (parallel and/or perpendicular lines) [8.8.4.1; 8.8.4.2]

(a) If A(0;-4) , B(-5;-5) , C(1;4) and D(2;9)

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} \qquad m_{CD} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m_{AB} = \frac{-5-0}{-5+4} \qquad m_{CD} = \frac{9-4}{2-1}$$
$$m_{AB} = \frac{-5}{-1} \qquad m_{CD} = \frac{5}{1}$$
$$m_{AB} = 5 \qquad m_{CD} = 5$$

$$m_{AB} = m_{CD}$$
$$\therefore AB // CD$$

(b) If K(5;-4) , L(0;-8) , M(-1;-5) and N(3;-10)

$$m_{KL} = \frac{y_2 - y_1}{x_2 - x_1} \qquad m_{MN} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m_{KL} = \frac{-8+4}{0-5} \qquad m_{MN} = \frac{-10+5}{3+1}$$
$$m_{KL} = \frac{-4}{-5} \qquad m_{MN} = \frac{-5}{4}$$
$$m_{KL} = \frac{4}{5}$$

$$m_{KL} \times m_{MN} = -1$$
$$\therefore KL \perp MN$$

(c) If I(1;-5), J(-3;0), K(g;-4) and L(-3;1)

$$m_{IJ} = m_{KL}$$
$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$\frac{0+5}{-3-1} = \frac{1+4}{-3-g}$$
$$\frac{5}{-4} = \frac{5}{-3-g}$$
$$5(-3-g) = -20$$
$$-15-5g = -20$$
$$-5g = -5$$
$$\therefore g = 1$$

(d) G(1;-4) , H(0;-2) , I(3 ;q) and J(1;-1)

$$m_{GH} \times m_{IJ} = -1$$
$$\frac{-2+4}{0-1} \times \frac{-1-q}{1-3} = -1$$
$$\frac{2}{-1} \times \frac{-1-q}{-2} = -1$$
$$\frac{-1-q}{-2} = \frac{1}{2}$$
$$2(-1-q) = -2$$
$$-1-q = -1$$
$$-q = 0$$
$$\therefore q = 0$$

1.3 Midpoint theorem [8.8.2.1; 8.8.2.2; 8.8.2.3]

a) D(-10;-10) and E(5;5)

$$M(x;y) = (\frac{x_2 + x_1}{2}; \frac{y_2 + y_1}{2})$$
$$M(x;y) = (\frac{5-10}{2}; \frac{5-10}{2})$$
$$M(x;y) = (\frac{-5}{2}; \frac{-5}{2})$$

(b) J(2;-6) and K(-3;-4)

$$M(x;y) = (\frac{x_2 + x_1}{2}; \frac{y_2 + y_1}{2})$$
$$M(x;y) = (\frac{-3+2}{2}; \frac{-4-6}{2})$$
$$M(x;y) = (\frac{-1}{2}; \frac{-10}{2})$$
$$M(x;y) = (-\frac{1}{2}; -5)$$

(c) M(-1;-4) and N(-6;-2)

$$M(x;y) = (\frac{x_2 + x_1}{2}; \frac{y_2 + y_1}{2})$$
$$M(x;y) = (\frac{-6-1}{2}; \frac{-2-4}{2})$$
$$M(x;y) = (\frac{-7}{2}; \frac{-6}{2})$$
$$M(X;y) = (-\frac{7}{2}; -3)$$

(d) (-p-9k;6z-7m) and (-7p+7k;2z-7m)

$$M(x;y) = (\frac{x_2 + x_1}{2}; \frac{y_2 + y_1}{2})$$
$$M(x;y) = (\frac{-p-9k-7p+7k}{2}; \frac{6z-7m+2z-7m}{2})$$
$$M(x;y) = (\frac{-8p-2k}{2}; \frac{8z-14m}{2})$$
$$M(x;y) = (-4p-k; 4z-7m)$$

(e)

$$x = \frac{x_2 + x_1}{2}$$
$$1 = \frac{-4+q}{2}$$
$$2 = -4+q$$
$$q = 6$$

(f)

$$x = \frac{x_2 + x_1}{2}$$
$$5 = \frac{7 + y}{2}$$
$$10 = 7 + y$$
$$y = 3$$

(g)

$$y = \frac{y_2 + y_1}{2}$$
$$0 = \frac{-6 + p}{2}$$
$$0 = -6 + p$$
$$p = 6$$