GRADE 11_Equations of a line

11.8 Equations of a line

1. The equation of a line through two points

A. Determine the equations of the straight line through the following points:

(a) P(1;-1) and Q(-2;3)
(b) A(2;-5) and B(-3;-1)
(c) R(-4;-1) and S(-5;4)
(d) P(5;-4) and Q(4;-3)
(e) U(1;-1) and V(-3;-5)

B. Determine the equation of the line through one point and a given gradient.

(a) Gradient = -2 through (-5;4)
(b) Gradient = -3 through (-2;6)
(c) Gradient = $\frac{1}{2}$ through (4;-8)
(d) Gradient = 3 through (0;9)
(e) Gradient = -1 through (0;0)

2. Parallel and perpendicular lines

(a) Determine whether DE and FG are parallel or perpendicular if D(5;-5), E(4;-8), F(0;-2) and G(3;-1) are given.

(b) Determine whether AB and CD are parallel or perpendicular if A(-3;-4), B(-8;1), C(-4;-4) and D(-9;1) are given.

(c) Determine whether LM and NF are parallel or perpendicular if L(0;1), M(1;5), N(-1;4) and F(-5;5) are given.

(d) Calculate the value of y if DE$\perp$FG and D(-5;-2), E(-1;-1), F(-2;y) and G(-3;5) are given.

(e) Calculate the value of p if FG//HI and F(-4;0), G(-8;-5), H(p;-2) and I(-6;-7) are given.

Memo

1. The equation of a line through two points [8.8.5.1; 8.8.5.2]

A. Equation of the straight line.

(a) P(1;-1) and Q(-2;3)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{3-(-1)}{-2-(1)}$$

$$m = \frac{4}{-3}$$

$$y = mx + c$$

$$y = -\frac{4}{3}x + c$$

$$-1 = -\frac{4}{3}(1) + c$$

$$c = \frac{1}{3}$$

$$\therefore y = -\frac{4}{3}x + \frac{1}{3}$$

(b) A(2;-5) and B(-3;-1)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-1-(-5)}{-3-(2)}$$

$$m = \frac{4}{-5}$$

$$y = mx + c$$

$$y = -\frac{4}{5}x + c$$

$$-5 = -\frac{4}{5}(2) + c$$

$$c = -\frac{17}{5}$$

$$\therefore y = -\frac{4}{5}x - \frac{17}{5}$$

(c) R(-4;-1) and S(-5;4)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{4-(-1)}{-5-(-4)}$$

$$m = \frac{5}{-1} = -5$$

$$y = mx + c$$

$$y = -5x + c$$

$$-1 = -5(-4) + c$$

$$c = -21$$

$$\therefore y = -5x - 21$$

(d) P(5;-4) and Q(4;-3)

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad y = mx + c$$
$$m = \frac{-3-(-4)}{4-(5)} \qquad y = -1x + c$$
$$m = \frac{1}{-1} = -1 \qquad -4 = -(5) + c$$
$$c = 1$$

$$\therefore y = -x + 1$$

(e) U(1;-1) and V(-3;-5)

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad y = mx + c$$
$$m = \frac{-5-(-1)}{-3-(1)} \qquad y = 1x + c$$
$$m = \frac{-4}{-4} = 1 \qquad -1 = (1) + c$$
$$c = -2$$

$$\therefore y = x - 2$$

B. Lines with a given gradient and one point.

(a) Gradient = -2 through (-5;4)

$$y = mx + c$$
$$y = -2x + c$$
$$4 = -2(-5) + c$$
$$c = -6$$
$$\therefore y = -2x - 6$$

(b) Gradient = -3 through (-2;6)

$y = mx + c$

$y = -3x + c$

$6 = -3(-2) + c$

$c = 0$

$\therefore y = -3x$

(c) Gradient = $\frac{1}{2}$ through (4;-8)

$y = mx + c$

$y = \frac{1}{2}x + c$

$-8 = \frac{1}{2}(4) + c$

$c = -10$

$\therefore y = \frac{1}{2}x - 10$

(d) Gradient = 3 through (0;9)

$y = mx + c$

$y = 3x + c$

$9 = 3(0) + c$

$c = 9$

$\therefore y = 3x + 9$

(e) Gradient = -1 through (0;0)

$y = mx + c$

$y = -1x + c$

$0 = -1(0) + c$

$c = 0$

$\therefore y = -x$

2. Parallel and perpendicular lines [8.8.4.1; 8.8.4.2]

(a) Determine whether DE and FG are parallel or perpendicular if D(5;-5), E(4;-8), F(0;-2) and G(3;-1) are given.

$$m_{DE} = \frac{-8-(-5)}{4-5} = \frac{-3}{-1} = 3$$

$$m_{FG} = \frac{-1-(-2)}{3-0} = \frac{1}{3}$$

$m_{DE} \neq m_{FG}$, lines are not parallel.

$m_{DE} \times m_{FG} \neq -1$, lines are not perpendicular.

(b) Determine whether AB and CD are parallel or perpendicular if A(-3;-4), B(-8;1), C(-4;-4) and D(-9;1) are given.

$$m_{AB} = \frac{1-(-4)}{-8-(-3)} = \frac{5}{-5} = -1$$

$$m_{CD} = \frac{1-(-4)}{-9-(-4)} = \frac{5}{-5} = -1$$

$m_{AB} = m_{CD}$, lines are parallel.

(c) Determine whether LM and NF are parallel or perpendicular if L(0;1), M(1;5), N(-1;4) and F(-5;5) are given.

$$m_{LM} = \frac{5-1}{1-0} = \frac{4}{1} = 4$$

$$m_{NF} = \frac{5-4}{-5-(-1)} = \frac{1}{-4}$$

$m_{LM} \times m_{NF} = 4 \times \frac{-1}{4} = -1$, lines are perpendicular.

(d) Calculate the value of y if DE $\perp$ FG and D(-5;-2), E(-1;-1), F(-2;y) and G(-3;5) are given.

$$m_{DE} = \frac{-1-(-2)}{-1-(-5)} = \frac{1}{4}$$

$$m_{FG} = \frac{5-y}{-3-(-2)} = \frac{5-y}{-1} = -5+y$$

$DE \perp FG$:

$$\frac{1}{4}\times(-5+y)=-1$$
$$-5+y=-4$$
$$\therefore y=1$$

(e) Calculate the value of p if FG//HI and F(-4;0), G(-8;-5), H(p;-2) and I(-6;-7) are given.

$$m_{FG}=\frac{-5-0}{-8-(-4)}=\frac{-5}{-4}=\frac{5}{4}$$

$$m_{HI}=\frac{-7-(-2)}{-6-p}=\frac{-5}{-6-p}$$

FG//HI:

$$\frac{5}{4}=\frac{-5}{-6-p}$$
$$5(-6-p)=-20$$
$$-30-5p=-20$$
$$-5p=10$$
$$\therefore p=-2$$