



CAMI Mathematics: Grade 11

GRADE 11 Trigonometric identities

11.9 Trigonometric identities

1. Simplify the following identities

(a) $\cos^2 \alpha \left(\frac{1}{\cos^2 \alpha} - 1 \right)$

(b) $\frac{1}{\cos^2 \beta} (1 - \sin^2 \beta)$

(c) $\frac{1}{\cos^2 \alpha} \left(\frac{1}{\sin^2 \alpha} - 1 \right)$

(d) $\sin^2 \theta \left(1 + \frac{\cos^2 \theta}{\sin^2 \theta} \right)$

(e) $\tan^2 \theta \left(\frac{1}{\sin^2 \theta} - 1 \right)$

2. Proof the following identities

(a) $\tan^2 \pi - \sin^2 \pi = \tan^2 \pi \cdot \sin^2 \pi$

(b) $1 - \sin^4 \beta = \cos^2 \beta (1 + \sin^2 \beta)$

(c) $\sin^4 \alpha - \cos^4 \alpha = \sin^2 \alpha - \cos^2 \alpha$

(d) $\tan \beta \cdot \cos \beta = \sin \beta$

(e) $\frac{\cos \theta}{\sin \theta} \cdot \frac{1}{\cos^2 \theta} \cdot \sin^2 \theta = \tan \theta$

(e) $\sin^2 \alpha (1 + \tan^2 \alpha) = \tan^2 \alpha$

(f) $1 + \frac{\cos^3 \beta}{\sin^3 \beta} = \left(1 + \frac{\cos \beta}{\sin \beta} \right) \left(\frac{1}{\sin^2 \beta} - \frac{\cos \beta}{\sin \beta} \right)$



MEMO

1. Simplify the following identities [7.5.2.1]

$$\begin{aligned} \text{(a)} \quad & \cos^2 \alpha \left(\frac{1}{\cos^2 \alpha} - 1 \right) \\ &= \cos^2 \alpha \left(\frac{1 - \cos^2 \alpha}{\cos^2 \alpha} \right) \\ &= \cos^2 \alpha \left(\frac{\sin^2 \alpha}{\cos^2 \alpha} \right) \\ &= \sin^2 \alpha \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad & \frac{1}{\cos^2 \beta} (1 - \sin^2 \beta) \\ &= \frac{1}{\cos^2 \beta} (\cos^2 \beta) \\ &= 1 \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad & \frac{1}{\cos^2 \alpha} \left(\frac{1}{\sin^2 \alpha} - 1 \right) \\ &= \frac{1}{\cos^2 \alpha} \left(\frac{1 - \sin^2 \alpha}{\sin^2 \alpha} \right) \\ &= \frac{1}{\cos^2 \alpha} \left(\frac{\cos^2 \alpha}{\sin^2 \alpha} \right) \\ &= \frac{1}{\sin^2 \alpha} \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad & \sin^2 \theta \left(1 + \frac{\cos^2 \theta}{\sin^2 \theta} \right) \\ &= \sin^2 \theta \left(\frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta} \right) \\ &= \sin^2 \theta + \cos^2 \theta \\ &= 1 \end{aligned}$$



CAMI Mathematics: Grade 11

(e) $\tan^2 \theta \left(\frac{1}{\sin^2 \theta} - 1 \right)$

$$= \frac{\sin^2 \theta}{\cos^2 \theta} \left(\frac{1 - \sin^2 \theta}{\sin^2 \theta} \right)$$

$$= \frac{\sin^2 \theta}{\cos^2 \theta} \cdot \frac{\cos^2 \theta}{\sin^2 \theta}$$

$$= 1$$

2. Proof the following identities [7.5.3.1; 7.5.3.3]

(a) $\tan^2 \pi - \sin^2 \pi = \tan^2 \pi \cdot \sin^2 \pi$

LHS:

$$\tan^2 \pi - \sin^2 \pi$$

$$= \frac{\sin^2 \pi}{\cos^2 \pi} - \sin^2 \pi$$

$$= \frac{\sin^2 \pi - \sin^2 \pi \cdot \cos^2 \pi}{\cos^2 \pi}$$

$$= \frac{\sin^2 \pi (1 - \cos^2 \pi)}{\cos^2 \pi}$$

$$= \frac{\sin^2 \pi}{\cos^2 \pi} \cdot \frac{\sin^2 \pi}{1}$$

$$= \tan^2 \pi \cdot \sin^2 \pi$$

$$= RHS$$

(b) $1 - \sin^4 \beta = \cos^2 \beta (1 + \sin^2 \beta)$

LHS:

$$1 - \sin^4 \beta$$

$$= (1 - \sin^2 \beta)(1 + \sin^2 \beta)$$

$$= \cos^2 \beta (1 + \sin^2 \beta)$$

$$= RHS$$

(c) $\sin^4 \alpha - \cos^4 \alpha = \sin^2 \alpha - \cos^2 \alpha$

LHS:



CAMI Mathematics: Grade 11

$$\begin{aligned} & \sin^4 \alpha - \cos^4 \alpha \\ &= (\sin^2 \alpha + \cos^2 \alpha)(\sin^2 \alpha - \cos^2 \alpha) \\ &= 1 \cdot (\sin^2 \alpha - \cos^2 \alpha) \\ &= RHS \end{aligned}$$

(d) $\tan \beta \cdot \cos \beta = \sin \beta$

LHS:

$$\begin{aligned} & \tan \beta \cdot \cos \beta \\ &= \frac{\sin \beta}{\cos \beta} \cdot \cos \beta \\ &= \sin \beta \\ &= RHS \end{aligned}$$

(e) $\frac{\cos \theta}{\sin \theta} \cdot \frac{1}{\cos^2 \theta} \cdot \sin^2 \theta = \tan \theta$

LHS:

$$\begin{aligned} & \frac{\cos \theta}{\sin \theta} \cdot \frac{1}{\cos^2 \theta} \cdot \frac{\sin^2 \theta}{1} \\ &= \frac{\sin \theta}{\cos \theta} \\ &= \tan \theta \\ &= RHS \end{aligned}$$

(e) $\sin^2 \alpha (1 + \tan^2 \alpha) = \tan^2 \alpha$

LHS:

$$\begin{aligned} & \sin^2 \alpha \left(1 + \frac{\sin^2 \alpha}{\cos^2 \alpha} \right) \\ &= \sin^2 \alpha \left(\frac{\cos^2 \alpha + \sin^2 \alpha}{\cos^2 \alpha} \right) \\ &= \sin^2 \alpha \cdot \frac{1}{\cos^2 \alpha} \\ &= \frac{\sin^2 \alpha}{\cos^2 \alpha} \\ &= \tan^2 \alpha \\ &= RHS \end{aligned}$$



CAMI Mathematics: Grade 11

$$(f) \quad 1 + \frac{\cos^3 \beta}{\sin^3 \beta} = \left(1 + \frac{\cos \beta}{\sin \beta}\right) \left(\frac{1}{\sin^2 \beta} - \frac{\cos \beta}{\sin \beta}\right)$$

LHS:

$$1 + \frac{\cos^3 \beta}{\sin^3 \beta}$$

$$= \left(1 + \frac{\cos \beta}{\sin \beta}\right) \left(1 - \frac{\cos \beta}{\sin \beta} + \frac{\cos^2 \beta}{\sin^2 \beta}\right)$$

$$= \left(1 + \frac{\cos \beta}{\sin \beta}\right) \left(\frac{\sin^2 \beta + \cos^2 \beta}{\sin^2 \beta} - \frac{\cos \beta}{\sin \beta}\right)$$

$$= \left(1 + \frac{\cos \beta}{\sin \beta}\right) \left(\frac{1}{\sin^2 \beta} - \frac{\cos \beta}{\sin \beta}\right)$$

$$= RHS$$

