GRADE 11_Trigonometric identities

11.9 Trigonometric identities

1. Simplify the following identities

(a) $\cos^2\alpha(\frac{1}{\cos^2\alpha}-1)$

(b) $\frac{1}{\cos^2\beta}(1-\sin^2\beta)$

(c) $\frac{1}{\cos^2\alpha}(\frac{1}{\sin^2\alpha}-1)$

(d) $\sin^2\theta(1+\frac{\cos^2\theta}{\sin^2\theta})$

(e) $\tan^2\theta(\frac{1}{\sin^2\theta}-1)$

2. Proof the following identities

(a) $\tan^2\pi-\sin^2\pi=\tan^2\pi.\sin^2\pi$

(b) $1-\sin^4\beta=\cos^2\beta(1+\sin^2\beta)$

(c) $\sin^4\alpha-\cos^4\alpha=\sin^2\alpha-\cos^2\alpha$

(d) $\tan\beta.\cos\beta=\sin\beta$

(e) $\frac{\cos\theta}{\sin\theta}\cdot\frac{1}{\cos^2\theta}\cdot\sin^2\theta=\tan\theta$

(e) $\sin^2\alpha(1+\tan^2\alpha)=\tan^2\alpha$

(f) $1+\frac{\cos^3\beta}{\sin^3\beta}=(1+\frac{\cos\beta}{\sin\beta})(\frac{1}{\sin^2\beta}-\frac{\cos\beta}{\sin\beta})$

MEMO

1. Simplify the following identities [7.5.2.1]

(a) $\cos^2\alpha(\frac{1}{\cos^2\alpha}-1)$

$=\cos^2\alpha(\frac{1-\cos^2\alpha}{\cos^2\alpha})$

$=\cos^2\alpha(\frac{\sin^2\alpha}{\cos^2\alpha})$

$=\sin^2\alpha$

(b) $\frac{1}{\cos^2\beta}(1-\sin^2\beta)$

$=\frac{1}{\cos^2\beta}(\cos^2\beta)$

$=1$

(c) $\frac{1}{\cos^2\alpha}(\frac{1}{\sin^2\alpha}-1)$

$=\frac{1}{\cos^2\alpha}(\frac{1-\sin^2\alpha}{\sin^2\alpha})$

$=\frac{1}{\cos^2\alpha}(\frac{\cos^2\alpha}{\sin^2\alpha})$

$=\frac{1}{\sin^2\alpha}$

(d) $\sin^2\theta(1+\frac{\cos^2\theta}{\sin^2\theta})$

$=\sin^2\theta(\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta})$

$=\sin^2\theta+\cos^2\theta$

$=1$

(e) $\tan^2\theta(\frac{1}{\sin^2\theta}-1)$

$$=\frac{\sin^2\theta}{\cos^2\theta}(\frac{1-\sin^2\theta}{\sin^2\theta})$$
$$=\frac{\sin^2\theta}{\cos^2\theta}\cdot\frac{\cos^2\theta}{\sin^2\theta}$$
$$=1$$

2. Proof the following identities [7.5.3.1; 7.5.3.3]

(a) $\tan^2\pi-\sin^2\pi=\tan^2\pi.\sin^2\pi$

LHS:

$$\tan^2\pi-\sin^2\pi$$
$$=\frac{\sin^2\pi}{\cos^2\pi}-\sin^2\pi$$
$$=\frac{\sin^2\pi-\sin^2\pi.\cos^2\pi}{\cos^2\pi}$$
$$=\frac{\sin^2\pi(1-\cos^2\pi)}{\cos^2\pi}$$
$$=\frac{\sin^2\pi}{\cos^2\pi}\cdot\frac{\sin^2\pi}{1}$$
$$=\tan^2\pi.\sin^2\pi$$
$$=RHS$$

(b) $1-\sin^4\beta=\cos^2\beta(1+\sin^2\beta)$

LHS:

$$1-\sin^4\beta$$
$$=(1-\sin^2\beta)(1+\sin^2\beta)$$
$$=\cos^2\beta(1+\sin^2\beta)$$
$$=RHS$$

(c) $\sin^4\alpha-\cos^4\alpha=\sin^2\alpha-\cos^2\alpha$

LHS:

$\sin^4\alpha - \cos^4\alpha$

$= (\sin^2\alpha + \cos^2\alpha)(\sin^2\alpha - \cos^2\alpha)$

$= 1 \cdot (\sin^2\alpha - \cos^2\alpha)$

$= RHS$

(d) $\tan\beta.\cos\beta = \sin\beta$

LHS:

$\tan\beta \cdot \cos\beta$

$= \dfrac{\sin\beta}{\cos\beta} \cdot \cos\beta$

$= \sin\beta$

$= RHS$

(e) $\dfrac{\cos\theta}{\sin\theta} \cdot \dfrac{1}{\cos^2\theta} \cdot \sin^2\theta = \tan\theta$

LHS:

$\dfrac{\cos\theta}{\sin\theta} \cdot \dfrac{1}{\cos^2\theta} \cdot \dfrac{\sin^2\theta}{1}$

$= \dfrac{\sin\theta}{\cos\theta}$

$= \tan\theta$

$= RHS$

(e) $\sin^2\alpha(1+\tan^2\alpha) = \tan^2\alpha$

LHS:

$\sin^2\alpha(1+\dfrac{\sin^2\alpha}{\cos^2\alpha})$

$= \sin^2\alpha(\dfrac{\cos^2\alpha + \sin\ \alpha}{\cos^2\alpha})$

$= \sin^2\alpha \cdot \dfrac{1}{\cos^2\alpha}$

$= \dfrac{\sin^2\alpha}{\cos^2\alpha}$

$= \tan^2\alpha$

$= RHS$

(f) $1+\frac{\cos^3\beta}{\sin^3\beta}=(1+\frac{\cos\beta}{\sin\beta})(\frac{1}{\sin^2\beta}-\frac{\cos\beta}{\sin\beta})$

LHS:

$1+\frac{\cos^3\beta}{\sin^3\beta}$

$=(1+\frac{\cos\beta}{\sin\beta})(1-\frac{\cos\beta}{\sin\beta}+\frac{\cos^2\beta}{\sin^2\beta})$

$=(1+\frac{\cos\beta}{\sin\beta})(\frac{\sin^2\beta+\cos^2\beta}{\sin^2\beta}-\frac{\cos\beta}{\sin\beta})$

$=(1+\frac{\cos\beta}{\sin\beta})(\frac{1}{\sin^2\beta}-\frac{\cos\beta}{\sin\beta})$

$=RHS$