



CAMI Mathematics: Grade 11

GRADE 11 Trigonometric identities

11.9 Trigonometric identities

1. Simplify the following expressions.

(a) $\frac{\cos^2 \alpha}{\sin^2 \alpha} \left(\frac{1}{\cos^2 \alpha} - 1 \right)$

(b) $\tan^2 \beta \left(1 + \frac{1}{\tan^2 \beta} \right)$

(c) $\frac{1}{\sin^2 \theta} \left(\frac{1}{\cos^2 \theta} - 1 \right)$

2. Prove that the following identities are true.

(a) $1 - \sin^4 \alpha = \cos^2 \alpha (1 + \sin^2 \alpha)$

(b) $\tan \lambda \cdot \cos \lambda = \sin \lambda$

(c) $\frac{\cos \alpha}{\sin \alpha} \cdot \frac{1}{\cos^2 \alpha} \cdot \sin^2 \alpha = \tan \alpha$

(d) $\frac{\cos^4 \beta}{\sin^4 \beta} + \frac{\cos^2 \beta}{\sin^2 \beta} = \frac{1}{\sin^4 \beta} - \frac{1}{\sin^2 \beta}$



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MEMO

1. Simplify the following expressions. [7.5.2.1]

(a)

$$\begin{aligned} & \frac{\cos^2 \alpha}{\sin^2 \alpha} \left(\frac{1}{\cos^2 \alpha} - 1 \right) \\ &= \frac{\cos^2 \alpha}{\sin^2 \alpha} \left(\frac{1 - \cos^2 \alpha}{\cos^2 \alpha} \right) \\ &= \frac{\cos^2 \alpha}{\sin^2 \alpha} \cdot \frac{\sin^2 \alpha}{\cos^2 \alpha} \\ &= 1 \end{aligned}$$

(b)

$$\begin{aligned} & \tan^2 \beta \left(1 + \frac{1}{\tan^2 \beta} \right) \\ &= \tan^2 \beta \left(\frac{\tan^2 \beta + 1}{\tan^2 \beta} \right) \\ &= \tan^2 \beta + 1 \end{aligned}$$

(c)

$$\begin{aligned} & \frac{1}{\sin^2 \theta} \left(\frac{1}{\cos^2 \theta} - 1 \right) \\ &= \frac{1}{\sin^2 \theta} \left(\frac{1 - \cos^2 \theta}{\cos^2 \theta} \right) \\ &= \frac{1}{\sin^2 \theta} \cdot \frac{\sin^2 \theta}{\cos^2 \theta} \\ &= \frac{1}{\cos^2 \theta} \end{aligned}$$

2. Prove that the following identities are true. [7.5.3.1; 7.5.3.3]

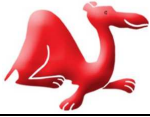
(a) $1 - \sin^4 \alpha = \cos^2 \alpha (1 + \sin^2 \alpha)$

LHS :

$$(1 + \sin^2 \alpha)(1 - \sin^2 \alpha)$$

$$= (1 + \sin^2 \alpha) \cdot \cos^2 \alpha$$

$$\therefore LHS = RHS$$



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(b)

$$\tan \lambda \cdot \cos \lambda = \sin \lambda$$

LHS :

$$\frac{\sin \lambda}{\cos \lambda} \cdot \cos \lambda$$

$$= \sin \lambda$$

$$\therefore LHS = RHS$$

(c)

$$\frac{\cos \alpha}{\sin \alpha} \cdot \frac{1}{\cos^2 \alpha} \cdot \sin^2 \alpha = \tan \alpha$$

LHS :

$$\frac{\cos \alpha}{\sin \alpha} \cdot \frac{1}{\cos \alpha} \cdot \frac{1}{\cos \alpha} \cdot \sin \alpha \cdot \sin \alpha$$

$$= \frac{\sin \alpha}{\cos \alpha}$$

$$= \tan \alpha$$

$$LHS = RHS$$

(d)

$$\frac{\cos^4 \beta}{\sin^4 \beta} + \frac{\cos^2 \beta}{\sin^2 \beta} = \frac{1}{\sin^4 \beta} - \frac{1}{\sin^2 \beta}$$

LHS :

$$\frac{\cos^4 \beta + \cos^2 \beta \cdot \sin^2 \beta}{\sin^4 \beta}$$

$$= \frac{\cos^2 \beta (\cos^2 \beta + \sin^2 \beta)}{\sin^4 \beta}$$

$$= \frac{\cos^2 \beta}{\sin^4 \beta}$$

RHS :

$$\frac{1}{\sin^4 \beta} - \frac{1}{\sin^2 \beta}$$

$$= \frac{1 - \sin^2 \beta}{\sin^4 \beta}$$

$$= \frac{\cos^2 \beta}{\sin^4 \beta}$$

$$LHS = RHS$$



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