CAMI Mathematics: Grade 11

GRADE 11_Trigonometric identities

11.9 Trigonometric identities

1. Simplify the following expressions.

(a) $\frac{\cos^2\alpha}{\sin^2\alpha}(\frac{1}{\cos^2\alpha}-1)$

(b) $\tan^2\beta(1+\frac{1}{\tan^2\beta})$

(c) $\frac{1}{\sin^2\theta}(\frac{1}{\cos^2\theta}-1)$

2. Prove that the following identities are true.

(a) $1-\sin^4\alpha=\cos^2\alpha(1+\sin^2\alpha)$

(b) $\tan\lambda\cdot\cos\lambda=\sin\lambda$

(c) $\frac{\cos\alpha}{\sin\alpha}\cdot\frac{1}{\cos^2\alpha}\cdot\sin^2\alpha=\tan\alpha$

(d) $\frac{\cos^4\beta}{\sin^4\beta}+\frac{\cos^2\beta}{\sin^2\beta}=\frac{1}{\sin^4\beta}-\frac{1}{\sin^2\beta}$

MEMO

1. Simplify the following expressions. [7.5.2.1]

(a)

$$\frac{\cos^2\alpha}{\sin^2\alpha}\left(\frac{1}{\cos^2\alpha}-1\right)$$
$$=\frac{\cos^2\alpha}{\sin^2\alpha}\left(\frac{1-\cos^2\alpha}{\cos^2\alpha}\right)$$
$$=\frac{\cos^2\alpha}{\sin^2\alpha}\cdot\frac{\sin^2\alpha}{\cos^2\alpha}$$
$$=1$$

(b)

$$\tan^2\beta\left(1+\frac{1}{\tan^2\beta}\right)$$
$$=\tan^2\beta\left(\frac{\tan^2\beta+1}{\tan^2\beta}\right)$$
$$=\tan^2\beta+1$$

(c)

$$\frac{1}{\sin^2\theta}\left(\frac{1}{\cos^2\theta}-1\right)$$
$$=\frac{1}{\sin^2\theta}\left(\frac{1-\cos^2\theta}{\cos^2\theta}\right)$$
$$=\frac{1}{\sin^2\theta}\cdot\frac{\sin_2\theta}{\cos^2\theta}$$
$$=\frac{1}{\cos^2\theta}$$

2. Prove that the following identities are true. [7.5.3.1; 7.5.3.3]

(a) $1-\sin^4\alpha=\cos^2\alpha(1+\sin^2\alpha)$

$LHS:$
$$(1+\sin^2\alpha)(1-\sin^2\alpha)$$
$$=(1+\sin^2\alpha)\cdot\cos^2\alpha$$
$$\therefore LHS = RHS$$

(b)

$\tan\lambda\cdot\cos\lambda=\sin\lambda$

$LHS:$

$\dfrac{\sin\lambda}{\cos\lambda}\cdot\cos\lambda$

$=\sin\lambda$

$\therefore LHS = RHS$

(c)

$\dfrac{\cos\alpha}{\sin\alpha}\cdot\dfrac{1}{\cos^2\alpha}\cdot\sin^2\alpha=\tan\alpha$

$LHS:$

$\dfrac{\cos\alpha}{\sin\alpha}\cdot\dfrac{1}{\cos\alpha}\cdot\dfrac{1}{\cos\alpha}\cdot\sin\alpha.\sin\alpha$

$=\dfrac{\sin\alpha}{\cos\alpha}$

$=\tan\alpha$

$LHS = RHS$

(d)

$\dfrac{\cos^4\beta}{\sin^4\beta}+\dfrac{\cos^2\beta}{\sin^2\beta}=\dfrac{1}{\sin^4\beta}-\dfrac{1}{\sin^2\beta}$

$LHS:$

$\dfrac{\cos^4\beta+\cos^2\beta\cdot\sin^2\beta}{\sin^4\beta}$

$=\dfrac{\cos^2\beta(\cos^2\beta+\sin^2\beta)}{\sin^4\beta}$

$=\dfrac{\cos^2\beta}{\sin^4\beta}$

$RHS:$

$\dfrac{1}{\sin^4\beta}-\dfrac{1}{\sin^2\beta}$

$=\dfrac{1-\sin^2\beta}{\sin^4\beta}$

$=\dfrac{\cos^2\beta}{\sin^4\beta}$

$LHS = RHS$

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