CAMI Mathematics: Grade 12

12.2 Patterns, sequences and series

12.2 Geometric series

1. Geometric series

(a) The sum of the first n terms of the geometric series -5 + 10 – 20 ….
is 25. Determine the number of terms in the series.

(b) Determine the sum of the first 3 terms of a geometric series with $T_1 = -4$ and $T_6 = 12500$.

(c) In a geometric series $T_1 = 1$, $T_n = -3125$ and the sum to n terms is -2604.
Determine the constant ratio and the number of terms.

(d) Determine the sum of the first 6 terms in a geometric series with $T_1 = 3$ and $T_4 = 192$.

2. Infinite geometric series

(a) Consider the series $\frac{1}{3} + \frac{1}{6} + \frac{1}{12} + ...$

(i) Does the series converge? Give a reason.
(ii) Calculate the sum to infinity.

(b) The first term in a geometric series is 14 and the sum to infinity is 56.
Determine the constant ratio.

(c) The constant ratio of a certain geometric series is $\frac{1}{3}$ and the sum to infinity is 18. Determine the first term in the series.

(d) Does the series 4 – 8 + 16 - ….. converge? Give a reason.

3. Addition problems

(a) The sum of terms 2, 3 and 4 is a geometric series equal to -56.
The sum of terms 4, 5 and 6 is -224. Determine a and $r\,(r > 0)$

(b) In a geometric series, T_3 is 100 more than T_2, and the sum of T_2 and T_3 is 60. Calculate a and r $(r < 0)$

(c) An athlete ran 306 km in the past 6 weeks to practice. He ran 41 km in the First week, and increased the weekly distance by a constant factor. Determine this constant factor.

MEMO

1. Geometric series [5.6.3]

(a) $S_n = 25$; $a = -5$; $r = -2$

$$25 = \frac{-5((-2)^n - 1)}{(-2-1)}$$

$$-75 = -5((-2)^n - 1)$$

$$15 = (-2)^n - 1$$

$$16 = (-2)^n$$

$$\therefore n = 4$$

(b) $T_1 = -4 = a$

$$T_6 = ar^5$$

$$12500 = (-4)r^5$$

$$-3125 = r^5$$

$$(-5)^5 = r^5$$

$$\therefore r = -5$$

$$S_n = \frac{a(r^n - 1)}{r-1}$$

$$S_3 = \frac{-4((-5)^3 - 1)}{(-5-1)}$$

$$S_3 = -84$$

(c) $T_1 = 1$, $T_n = -3125$, $S_n = -2604$

$$T_n = 1.r^{n-1}$$

$$-3125 = r^{n-1}$$

$$-3125r = r^n$$

$$-2604 = \frac{1(r^n - 1)}{r-1}$$

$$-2604r + 2604 = r^n - 1$$

$$-2604r + 2604 = -3125r - 1$$

$$\therefore r = 5$$

$$-3125(5) = (5)^n$$

$$\log 15625 = n \log 5$$

$$\therefore n = 6$$

(d) $T_1 = 3 = a$ and $T_4 = 192$

$T_4 = ar^3$

$192 = 3.r^3$

$64 = r^3$

$\therefore r = 4$

$S_6 = \frac{3(4^5 - 1)}{4 - 1}$

$S_6 = 1023$

2. Infinite geometric series [5.6.4]

(a) $\frac{1}{3} + \frac{1}{6} + \frac{1}{12} + \ldots\ldots$

(i) $r = \frac{1}{2}$ thus $-1 < r < 1$ so the series will converge.

(ii) $a = \frac{1}{3}$ and $r = \frac{1}{2}$

$S_\infty = \frac{a}{1 - r}$

$S_\infty = \frac{1}{3} \div \frac{1}{2}$

$S_\infty = \frac{2}{3}$

(b) $a = 14$ and $S_\infty = 56$

$S_\infty = \frac{a}{1 - r}$

$56 = \frac{14}{1 - r}$

$56 - 56r = 14$

$-56r = -42$

$\therefore r = \frac{3}{4}$

(c) $r = \frac{1}{3}$

$S_\infty = \frac{a}{1-r}$

$18 = \frac{a}{1-\frac{1}{3}}$

$18 = \frac{a}{\frac{2}{3}}$

$\therefore a = 12$

(d) 4 – 8 + 16 - …..

$r = \frac{-8}{4} = -2$

Does not converge because $r \notin (-1;1)$

3. Addition problems [5.6.5; 5.6.6]

(a) $T_2 + T_3 + T_4 = -56$

$ar + ar^2 + ar^3 = -56$

$ar(1 + r + r^2) = -56$

$T_4 + T_5 + T_6 = -224$

$ar^3 + ar^4 + ar^5 = -224$

$ar^3(1 + r + r^2) = -224$

$\frac{ar^3(1+r+r^2)}{ar(1+r+r^2)} = \frac{-224}{-56}$

$\therefore r^2 = 4$

$\therefore r = 2$

$ar(1 + r + r^2) = -56$

$2a(1 + 2 + 4) = -56$

$2a = 8$

$\therefore a = 4$

(b) $T_3 = 100 + T_2$

$ar^2 = 100 + ar$

$ar^2 - ar = 100$

$ar(r-1) = 100$

$T_2 + T_3 = 60$

$ar + ar^2 = 60$

$ar(1 + r) = 60$

$\frac{ar(r-1)}{ar(1+r)} = \frac{100}{60}$

$6(r-1) = 10(1+r)$

$6r - 6 = 10 + 10r$

$-4r = 16$

$\therefore r = -4$

$ar + ar^2 = 60$

$-4a(1-4) = 60$

$12a = 60$

$\therefore a = 5$

(c) $S_6 = 306$, $a = 41$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$
$$306 = \frac{6}{2}(2(41) + 5d)$$
$$102 = 82 + 5d$$
$$20 = 5d$$
$$\therefore d = 4$$