12.5 Calculus

12.5 Practical application

A. Application on area, volume and perimeter

1. Calculate the dimensions of a rectangle with a perimeter of 312 m for which the area, V, is at a maximum.

2. A rectangle's width and height, when added, are 114mm. Determine the width and the height when the area, P, is at a maximum.

3. The height of an isosceles triangle is (18 – x)mm and the base is (20 + 2x)mm. Determine x for a maximum area.

4. The dimensions of a prism is b cm, b cm and $(60-4b)$cm. Determine b if the Volume, V, is a maximum.

5. The area of a rectangle is 64m². Calculate the length of the sides if the perimeter is at a minimum.

B. Advanced problems on maxima and minima

1. A ball is kicked straight up in the air. If its height after t seconds is $h(t)=276t-6t^2$, calculate the maximum height the ball will attain.

2. A livestock dealer buys 47 sheep at $R138$ each. If she sells the sheep at $R138+6x$ each after losing x sheep, calculate the profit $(P(x))$ if is was a maximum.

3. In order to protect his vegetables, a farmer buys 1748m of fencing. Give the dimensions of the vegetable garden if it should cover a maximum area.

MEMO

A. Application on area, volume and perimeter [5.7.3.3; 5.7.3.4]

1. $Perimeter = 2x + 2y$ and the $Area = xy$

$$Perimeter = 2x + 2y$$
$$312 = 2x + 2y$$
$$156 = x + y$$
$$\therefore y = 156 - x$$

$$\therefore y = 78$$

$$Area = xy$$
$$V = x(156 - x)$$
$$V = 156x - x^2$$
$$\frac{dV}{dx} = 156 - 2x$$
$$0 = 156 - 2x$$
$$x = 78$$

2. $Perimeter = 2x + 2y$ and $Area = xy$

$$114 = x + y$$
$$\therefore y = 114 - x$$

$$\therefore y = 57$$

$$Area = xy$$
$$P = x(114 - x)$$
$$P = 114x - x^2$$
$$\frac{dP}{dx} = 114 - 2x$$
$$0 = 114 - 2x$$
$$\therefore x = 57$$

3. $Area = \frac{1}{2} \times base \times height$

$$A = \frac{1}{2}(20 + 2x)(18 - x)$$
$$A = -x^2 + 8x + 180$$
$$Maximum:$$
$$\frac{dA}{dx} = -2x + 8$$
$$0 = -2x + 8$$
$$\therefore x = 4$$

4. $Volume = length \times breadth \times height$

$Volume = b \times b \times (6 - 4b)$

$V = b^2(60 - 4b)$

$V = 60b^2 - 4b^3$

$Maximum:$

$\frac{dV}{dx} = 120b - 12b^2$

$0 = -12b(b - 10)$

$\therefore b = 10$

5. $Area = xy$ and $Perimeter = 2x + 2y$

$Perimeter = 2x + 2(\frac{64}{x})$

$P = 2x + \frac{128}{x}$

$P = 2x + 128x^{-1}$

$Minimum:$

$\frac{dP}{dx} = 2 - 128x^{-2}$

$0 = 2 - 128x^2$

$x^2 64$

$x = 8$

$64 = xy$

$\therefore y = \frac{64}{x}$

$\therefore y = 8$

B. Advanced problems on maxima and minima [5.7.3.5]

1. $h(t) = 276t - 6t^2$

$h'(x) = 276 - 12t$

$0 = 276 - 12t$

$t = 23\text{sec}$

$\max.height = 276(23) - 6(23)^2$

$\max.height = 3174m$

2. $P(x) = Num.sold \times price - \cos t$

$P(x) = (47 - x)(138 + 6x) - 47 \times R138$

$P(x) = 6486 + 144x - 6x^2 - 6486$

$P(x) = 144x - 6x^2$

$Max.profit:$

$\frac{dP}{dx} = 144 - 12x$

$0 = 144 - 12x$

$\therefore x = 12$

$Max.profit = 144(12) - 6(12)^2 = R864$

3. $Perimeter = 2x + 2y$ and $Area = xy$

$1748 = 2x + 2y$

$874 = x + y$

$\therefore y = 874 - x$

$\therefore y = 437$

$Area = x(874 - x)$

$A = 874x - x^2$

$Maximum:$

$\frac{dA}{dx} = 874 - 2x$

$0 = 874 - 21$

$\therefore x = 437$