12.9 Trigonometry
12.9 Compound angle identities

1. Introduction to compound angles

(a) sin (270° - α)
(b) cos (180° + π)
(c) sin (360° - ∂)
(d) cos (90° - Ω)
(e) cos (90° + π)

2. Simplify

(a) cos 12π.sin 9π – sin12π.cos 9π
(b) sin 20°.cos 10° + cos 20°.sin 10°
(c) -sin14α.sin 8α – cos 14α.cos 8α
(d) cos 65°.cos 35° + sin 65°.sin 35°
(e) cos 7∂.sin 3∂ + sin 7∂.cos 3∂

3. Advanced simplification

(a) cos 55°.cos 350° + sin 55°.sin 10°
(b) sin 45°.cos 345° + cos 45°.sin 15°
(c) cos 83°.cos 322° + sin 83°.sin 38°
(d) sin 78°.cos342° - cos 78°.sin 18°
(e) sin 12°.cos 312° + cos 12°.sin 48°

MEMO

1. Introduction to compound angles [7.5.4.1]

(a) $\sin(270° - \alpha)$

$\sin(270° - \alpha) = \sin 270°.\cos\alpha - \cos 270°.\sin\alpha$
$\sin(270° - \alpha) = -1.\cos\alpha - 0.\sin\alpha$
$\sin(270° - \alpha) = -\cos\alpha$

(b) $\cos(180° + \pi)$

$\cos(180° + \pi) = \cos 180°.\cos\pi - \sin 180° \sin\pi$
$\cos(180° + \pi) = -1.\cos\pi - 0.\sin\pi$
$\cos(180° + \pi) = -\cos\pi$

(c) $\sin(360° - \partial)$

$\sin(360° - \partial) = \sin 360°.\cos\partial - \cos 360°.\sin\partial$
$\sin(360° - \partial) = 0.\cos\partial - 1.\sin\partial$
$\sin(360° - \partial) = -\sin\partial$

(d) $\cos(90° - \Omega)$

$\cos(90° - \Omega) = \cos 90°.\cos\Omega + \sin 90°.\sin\Omega$
$\cos(90° - \Omega) = 0.\cos\Omega + 1.\sin\Omega$
$\cos(90° - \Omega) = \sin\Omega$

(e) $\cos(90° + \pi)$

$\cos(90° + \pi) = \cos 90°.\cos\pi - \sin 90°.\sin\pi$
$\cos(90° + \pi) = 0.\cos\pi - 1.\sin\pi$
$\cos(90° + \pi) = -\sin\pi$

2. Simplify [7.5.4.2]

(a) $\cos 12\pi.\sin 9\pi - \sin 12\pi.\cos 9\pi$

$= \sin(9\pi - 12\pi)$

$= \sin(-3\pi)$

$= -\sin(3\pi)$

(b) $\sin 20°.\cos 10° + \cos 20°.\sin 10°$

$= \sin(20° + 10°)$

$= \sin 30°$

$= \frac{1}{2}$

(c) $-\sin 14\alpha.\sin 8\alpha - \cos 14\alpha.\cos 8\alpha$

$= -(\sin 14\alpha.\sin 8\beta + \cos 14\alpha.\cos 8\alpha)$

$= -\cos(14\alpha - 8\alpha)$

$= -\cos 6\alpha$

(d) $\cos 65°.\cos 35° + \sin 65°.\sin 35°$

$= \cos(65° - 35°)$

$= \cos 30°$

$= \frac{\sqrt{3}}{2}$

(e) $\cos 7\partial.\sin 3\partial + \sin 7\partial.\cos 3\partial$

$= \sin(3\partial + 7\partial)$

$= \sin 10\partial$

3. Advanced simplification [7.5.4.3]

(a) $\cos 55°.\cos 350° + \sin 55° \sin 10°$

$= \cos 55°.\cos 10° + \sin 55°.\sin 10°$

$= \cos(55° - 10°)$

$= \cos 45°$

$= \frac{\sqrt{2}}{2}$

(b) $\sin 45°.\cos 345° + \cos 45°.\sin 15°$

$= \sin 45°.\cos 15° + \cos 45°.\sin 15°$

$= \sin(45° + 15°)$

$= \sin 60°$

$= \dfrac{\sqrt{3}}{2}$

(c) $\cos 83°.\cos 322° + \sin 83°.\sin 38°$

$= \cos 83°.\cos 38° + \sin 83°.\sin 38°$

$= \cos(83° - 38°)$

$= \cos 45°$

$= \dfrac{\sqrt{2}}{2}$

(d) $\sin 78°.\cos 342° - \cos 78°.\sin 18°$

$= \sin 78°.\cos 18° - \cos 78°.\sin 18°$

$= \sin(78° - 18°)$

$= \sin 60°$

$= \dfrac{\sqrt{3}}{2}$

(e) $\sin 12°.\cos 312° + \cos 12°.\sin 48°$

$= \sin 12°.\cos 48° + \cos 12°.\sin 48°$

$= \sin(12° + 48°)$

$= \sin 60°$

$= \dfrac{\sqrt{3}}{2}$