ZERO of a function

In mathematics, a zero, also sometimes called a root, of a function is an x-value from the domain that will ensure that $f(x) = 0$. If a function maps real numbers, its zeros are the x-intercept and therefore the coordinate (x ; 0) belongs to the zero point of the function. Graphically this means that the zero point of a function is the x-intercept (where the graph meets the x-axis).

Example:

$$f(x) = x^2 - 6x + 9$$
$$f(3) = 3^2 - 6(3) + 9$$
$$f(3) = 0$$

Thus, 3 is a root for f(x).

Computing roots

In the school curriculum learners must be able to solve linear, quadratic and cubic functions.

Linear functions

The method for solving linear equations is based on arranging like terms on one side of the equal side. The four basic operations are needed to calculate the final answer. A linear equation will always have only one root.

Example: Solve for x if $2x + 4 = 3x - 2$.

$$2x + 4 = 3x - 2$$
$$2x - 3x = -2 - 4$$
$$-x = -6$$
$$\therefore x = 6$$

Quadratic equations

A quadratic equation is an equation involving a second power of an unknown. The general form is $ax^2 + bx + c = 0$.There is three ways to solve this kind of equation and the result will be two equal or unequal roots.

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METHODS

A. Factorization

The one side of the equation must be fully factorized and it must be equal to zero. The factors will be equaled to zero from where the zero points will be calculated.

Example: Solve for x if $x^2 + 7x + 12 = 0$.

$$x^2 + 7x + 12 = 0$$
$$(x + 4)(x + 3) = 0$$
$$\therefore x = -4 \text{ or } -3$$

B. Completing the square

$$ax^2 + bx + c = 0$$
$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$
$$x^2 + \frac{b}{a}x + (\frac{b}{2a})^2 = -\frac{c}{a} + (\frac{b}{2a})^2$$
$$(x + \frac{b}{2a})^2 = \frac{-4ac}{4a^2} + \frac{b^2}{4a^2}$$
$$x + \frac{b}{2a} = \sqrt{\frac{b^2 - 4ac}{4a^2}}$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

C. Quadratic formula

The equation $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ is called the quadratic formula that allows us to solve the quadratic equation by substituting a, b and c values as provided in the equations.

Cubic functions

A cubic equation is a polynomial equation, $ax^3 + bx^2 + cx + d = 0$, will produces three roots once solved. There are various methods to use when solving the cubic functions.

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METHODS

I. Derivative

Through the quadratic formula the roots of the derivative $f'(x) = 3ax^2 + 2bx + c$ are given by $x = \frac{-b \pm \sqrt{b^2 - 3ac}}{3a}$ and can be used to calculate the critical points where the slope of the cubic function will be zero. If $b^2 - 3ac > 0$, the cubic function has a local maximum and a local minimum if $b^2 - 3ac < 0$. There will be a point of inflection if $b^2 - 3ac = 0$.

II. Vieta's substitution

Starting from the depressed cubic $x^3 + px + q = 0$, we make the following substitution, known as Vieta's substitution:

$$x = w - \frac{p}{3w}$$

This results in the equation $w^3 + q - \frac{p^3}{27w^3} = 0$.

Multiplying by w^3, it becomes an equation in w to the power of six, which is in fact a quadratic equation in w^3:

$$w^6 + qw^3 - \frac{p^3}{27} = 0.$$

The quadratic formula allows us to solve it in w^3. If w_1, w_2 and w_3 are the three cubic roots of one of the solutions in w^3, then the roots of the original depressed cubic are:

$$x_1 = w_1 - \frac{p}{3w_1}, \quad x_2 = w_2 - \frac{p}{3w_2} \text{ and } x_3 = w_3 - \frac{p}{3w_3}$$

III. Other methods

- Cardano's method
- Lagrange's method
- Trigonometric (and hyperbolic) method

On school level the derivative method is used to solve cubic equations.

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REFERENCES

1. http://en.wikipedia.org/wiki/Cubic_equations#Cardano.27s_method

2. http://en.wikipedia.org/wiki/Root-finding_algorithm

3. **Dictionary of mathematic terms**