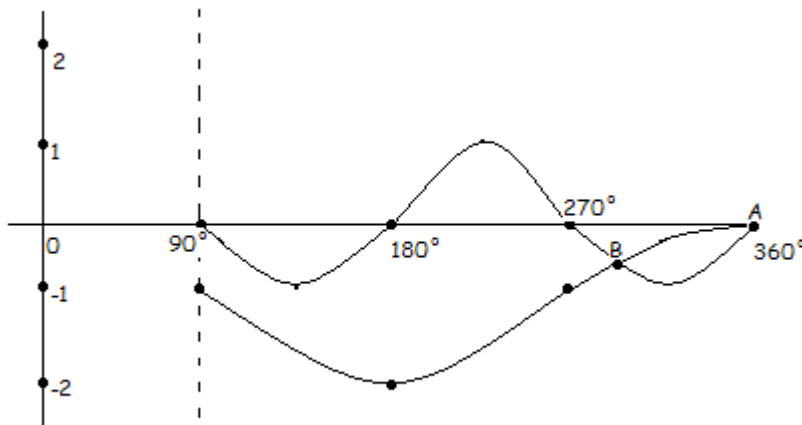


MEMO: Grade 12 Paper 2

1.1	$(x^2 + 6x + 9) + (y^2 - 2y + 1) + 1 = 10$ $(x + 3)^2 + (y - 1)^2 = 9$ $Q(-3 ; 1)$	✓✓ ✓✓	(4)
1.2	$B(-3 ; 4)$ and $C(-3 ; -2)$	✓✓	(2)
1.3	$P(x ; y) = (\frac{-3+0}{2}; \frac{4+1}{2}) = (\frac{-3}{2}; \frac{5}{2})$	✓✓	(2)
1.4	$BQ^2 = 9$ and $QC^2 = 9$ $PC^2 = (-3 + \frac{3}{2})^2 + (-2 - \frac{5}{2})^2 = \frac{90}{4}$ $BQ^2 - PC^2 = 9 - \frac{90}{4} = \frac{-27}{2}$ $PB^2 = (-3 + \frac{3}{2})^2 + (4 - \frac{5}{2})^2 = \frac{18}{4}$ $3(PB^2 - QC^2) = 3(\frac{18}{4} - 9) = \frac{-27}{2}$	✓ ✓ ✓ ✓ ✓	(5)
2.1	$A(2 ; 0)$ $AC = 4$ units $AB = \sqrt{(2-0)^2 + (0-2\sqrt{3})^2} = 4$ units $CB = \sqrt{(-2-0)^2 + (0-2\sqrt{3})^2} = 4$ units $AB = CB = AC$ Equilateral triangle	✓ ✓ ✓ ✓	(4)
2.2	$CD = 3OC = 3(2) = 6$ units $D(6 ; 0)$ and $B(0 ; 2\sqrt{3})$ $M_{DB} = \frac{2\sqrt{3}-0}{0-6} = -\frac{\sqrt{3}}{3}$ $\tan \theta = -\frac{\sqrt{3}}{3}$ $\theta = 150^\circ$	✓ ✓ ✓	(3)
2.3	$AB^2 = 16$ $DB^2 = (6-0)^2 + (0-2\sqrt{3})^2 = 48$ $3 \times AB^2 = 3 \times 16 = 48 = DB^2$	✓ ✓	(2)
3.1	$Q(-4 ; -4) ; P(8 ; 6)$	✓✓	(2)
3.2	$A(2 ; 6) ; D(-2 ; 0)$	✓✓	(2)
3.3	$X = 2 ; x = -2$	✓✓	(2)
3.4	$AB \parallel CD$ special lines $AB = 6$ units	✓	

$C(-2 ; -6)$ and $CD = 6$ units $AB = CD = 6$ units $ABCD$ a parallelogram, $[AB//CD, AB = CD]$	✓ ✓ ✓ (4)
4.1 $x^2 + y^2 = r^2$ $m_{\text{radius}} = \frac{b}{a}$ $m_{\text{tangent}} = \frac{-a}{b}$ Tangent: $y = \frac{-a}{b}x + c$ $by = -ax + bc$ In $\triangle COA$: $C(0 ; c)$ and $A(a ; b)$ $OC^2 = OA^2 + AC^2$ $c^2 = r^2 + (a - 0)^2 + (b - c)^2$ $c^2 = r^2 + a^2 + b^2 - 2bc + c^2$ $c^2 = r^2 + r^2 - 2bc + c^2$ $2bc = 2r^2$ $bc = r^2$ Tangent: $bx + ay - r^2 = 0$	✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ (6)
5.1 $3\sin x + 2 = 0 ; x \in [0^\circ ; 360^\circ]$ $\sin x = -\frac{2}{3}$, Ref Angle: $x = 41.81.03...^\circ$ 3rd quadrant: $x = 221.81^\circ$ 4th quadrant: $x = 318.19^\circ$	✓ (6) ✓✓ ✓ ✓ (4)
5.2.1 $\sin^2 \frac{x}{2} - 2\sin \frac{x}{2} - 3 = 0$ $(\sin \frac{x}{2} - 3)(\sin \frac{x}{2} + 1) = 0$ $\sin \frac{x}{2} \neq 3 ; \sin \frac{x}{2} = -1$ $\frac{x}{2} = 270^\circ + k.360^\circ$ $x = 540^\circ + k.720^\circ$	✓✓ ✓✓ ✓ ✓ (6)
5.2.2 $x = 540^\circ ; x = -180^\circ$	✓✓ (2)
6.1	✓✓ (2)



✓✓✓
✓✓✓

(6)

6.2 $x = 180^\circ$

6.3 $180^\circ < x < 360^\circ$

6.4 See graph

✓✓
✓✓
✓✓

(2)
(2)
(2)

7.1.1 $\cos^2 34^\circ + \sin^2 34^\circ = 1$
 $\sin^2 34^\circ = 1 - \cos^2 34^\circ$
 $\sin 34^\circ = \sqrt{1 - t^2}$

✓✓

(2)

7.1.2 $\sin 68^\circ = \sin 2(34^\circ) = 2\sin 34^\circ \cos 34^\circ$
 $= 2t\sqrt{1 - t^2}$

✓✓

(2)

7.1.3 $\tan^2 34^\circ = \frac{\sin^2 34^\circ}{\cos^2 34^\circ} = \frac{1 - t^2}{t^2}$

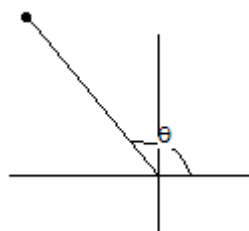
✓✓

(2)

7.2 $\tan \theta = -\frac{5}{2} = \frac{y}{x}; \theta \in [90^\circ; 270^\circ]$

2nd quadrant: $r = \sqrt{29}$

(-2;5)



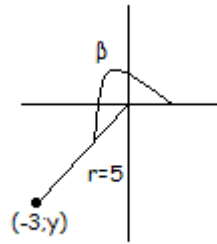
✓

✓

$5\cos \beta + 3 = 0; \cos \beta = \frac{-3}{5}$

3rd quadrant: $y = -4$

✓



$$\begin{aligned}\sin(\theta + \beta) &= \sin \theta \cos \beta + \cos \theta \sin \beta \\ &= \frac{5}{\sqrt{29}} \cdot \frac{-3}{5} + \frac{-2}{\sqrt{29}} \cdot \frac{-4}{5} \\ &= \frac{-7}{5\sqrt{29}}\end{aligned}$$

✓

✓

(5)

$$\begin{aligned}7.3 \quad \cos \frac{1}{2}x &= \cos \frac{1}{2}(210^\circ) = \cos 105^\circ \\ \cos(105^\circ) &= \cos(60^\circ + 45^\circ) \\ &= \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ \\ &= \frac{1}{2} \cdot \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} \\ &= \frac{1 - \sqrt{3}}{2\sqrt{2}}\end{aligned}$$

✓

✓✓

✓

✓

✓

(6)

$$\begin{aligned}&\frac{\sin \theta \cos^2 \theta - \sin^3 \theta}{2 \cos^3 \theta - \cos \theta} \\ &= \frac{\sin \theta (\cos^2 \theta - \sin^2 \theta)}{\cos \theta (2 \cos^2 \theta - 1)}\end{aligned}$$

✓✓

$$\begin{aligned}7.4.1 \quad \text{LHS:} &= \frac{\sin \theta \cos 2\theta}{\cos \theta \cos 2\theta} \\ &= \frac{\sin \theta}{\cos \theta} \\ &= \tan \theta \\ &= \text{RHS}\end{aligned}$$

✓

✓

(4)

7.4.2

$\frac{\sin^2 \theta + \cos^2 \theta}{\frac{1}{2}[\cos A \cos 60^\circ - \sin A \sin 60^\circ] + \frac{\sqrt{3}}{2}[\sin A \cos 60^\circ + \cos A \sin 60^\circ]}$ $= \frac{1}{\frac{1}{2}[\cos A \frac{1}{2} - \sin A \frac{\sqrt{3}}{2}] + \frac{\sqrt{3}}{2}[\sin A \frac{1}{2} + \cos A \frac{\sqrt{3}}{2}]}$ $= \frac{1}{\frac{\cos A}{4} - \frac{\sqrt{3} \sin A}{4} + \frac{\sqrt{3} \sin A}{4} + \frac{3 \cos A}{4}}$ $= \frac{1}{\cos A}$ $= RHS$	✓✓ ✓✓ ✓✓ (6)
8.1 $\hat{P} = 180^\circ - (\theta + \beta)$ Area $\triangle PQR = \frac{1}{2}pq \sin \beta$ $\frac{q}{\sin \theta} = \frac{p}{\sin(\theta + \beta)}$ $q = \frac{p \sin \theta}{\sin(\theta + \beta)}$ Area $\triangle PQR = \frac{1}{2}p \cdot \frac{p \sin \theta}{\sin(\theta + \beta)} \cdot \sin \beta$ $= \frac{p^2 \sin \theta \sin \beta}{2 \sin(\theta + \beta)}$	✓ ✓ ✓ ✓ (4)
8.2.1 $\frac{CE}{DC} = \frac{1}{\tan \alpha}$ and $\frac{EB}{AB} = \frac{1}{\tan \alpha}$ $CE = \frac{a}{\tan \alpha}$ and $EB = \frac{2a}{\tan \alpha}$	✓✓ ✓✓ (4)
8.2.2 $BC^2 = CE^2 + EB^2 - 2 \cdot BC \cdot EB \cdot \cos 120^\circ$ $= \left(\frac{a}{\tan \alpha}\right)^2 + \left(\frac{2a}{\tan \alpha}\right)^2 - 2\left(\frac{a}{\tan \alpha}\right)\left(\frac{2a}{\tan \alpha}\right)(-\cos 60^\circ)$ $= \frac{a^2}{\tan \alpha} + \frac{4a^2}{\tan \alpha} + \frac{4a^2}{\tan \alpha} \cdot \frac{1}{2}$ $= \frac{7a^2}{\tan \alpha}$ $BC = \frac{a\sqrt{7}}{\tan \alpha}$	✓ ✓

9.1.1 Median = 12

✓ (3)

9.1.2 $Q_2 = 12$

✓✓ (2)

$Q_1 = 9$

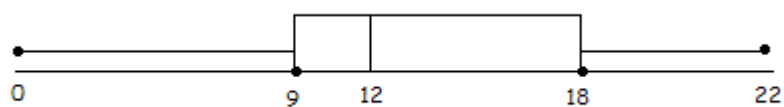
$Q_3 = 18$

✓
✓ (2)

9.1.3 $(Q_1 - 1,5IQR ; Q_3 + 1,5 IQR)$
 $= (9 - 1,5 \times 9 ; 18 + 1,5 \times 9)$
 $= (-4,5 ; 31,5)$
 no outliers

✓✓
✓✓ (4)

9.1.4



✓✓✓✓✓ (5)

9.2.1 $\bar{x} = 166,87$
 $\sigma = 13,176$

✓✓ (2)

9.2.2 $\bar{x}$ and σ will increase

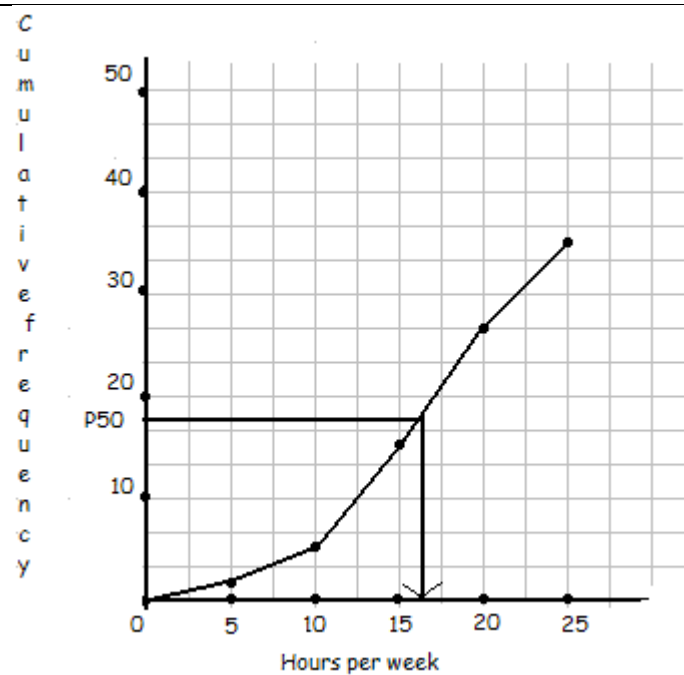
✓✓ (2)

9.3.1

Hours per week	Frequency	Cumulative frequency
$0 < x < 5$	1	1
$5 < x < 10$	4	5
$10 < x < 15$	10	15
$15 < x < 20$	12	27
$20 < x < 25$	8	35

✓✓ (2)

9.3.2



✓✓✓✓✓ (4)

✓✓ (2)

9.3.3 Median

10.1.1 $P(x; y) \rightarrow P'(-x; y)$

✓✓ (2)

10.1.2 $P'(-x; y) \rightarrow P''(-y; -x)$

✓✓ (2)

10.1.3 $P''(-y; -x) \rightarrow P'''(-y-2; -x-4)$

✓✓ (2)

10.1.4 $(-1; -7)$

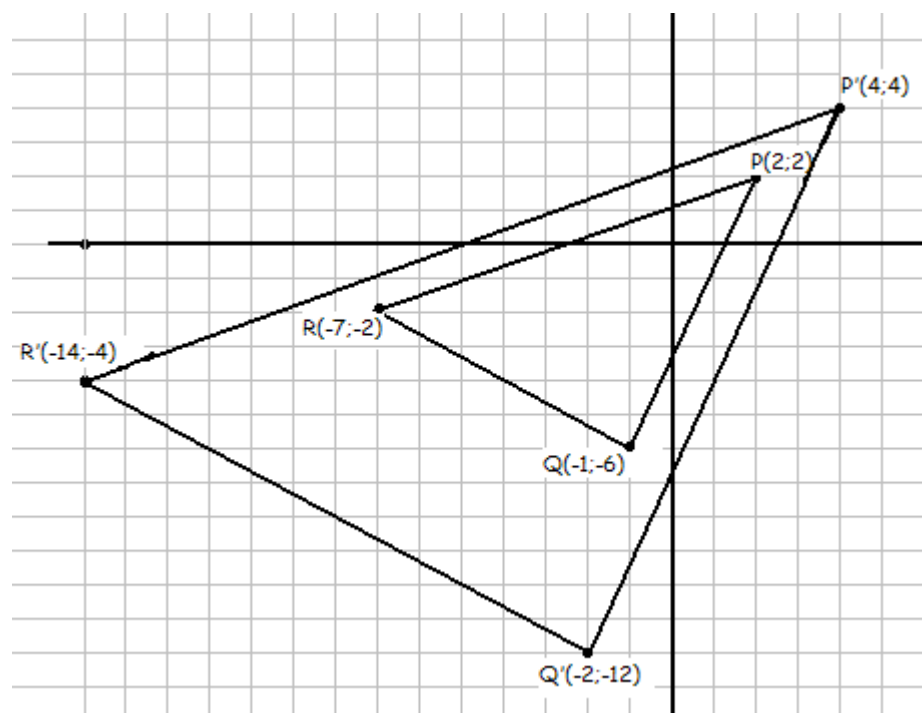
✓✓ (2)

10.2.1 $P(2; 2) \rightarrow P'(4; 4)$

✓✓ (2)

$Q(-1; -6) \rightarrow Q'(-2; -12)$

$R(-7; -2) \rightarrow R'(-14; -4)$



✓✓✓ (3)

10.2.2 Area $\Delta PQR = 30$, Area $\Delta P'Q'R' = 120$
 1 : 4

✓✓

(2)

10.3 $x' = -2 \cos(-150^\circ) + \sqrt{3} \sin(-150^\circ)$

$$x' = -2\cos 150^\circ - \sqrt{3} \sin(150^\circ)$$

$$x' = +2\cos 30^\circ - \sqrt{3} \sin 30^\circ$$

$$x' = 2 \cdot \frac{\sqrt{3}}{2} - \sqrt{3} \cdot \frac{1}{2}$$

$$x' = \frac{\sqrt{3}}{2}$$

✓

✓

✓

✓

(4)

10.4.1 rotation 180° around the origin

✓✓

(2)

10.4.2 $(x ; y) \rightarrow (-x ; -y)$

✓✓

(2)

10.4.3 $(-x ; -y) \rightarrow (-3x ; -3y)$

✓✓

(2)

TOTAL:150